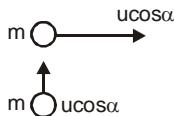


Physics

1. d $P = \rho \frac{RT}{M}$

$$\frac{\rho_1}{\rho_2} = \frac{P_1 M_1}{P_2 M_2} = \frac{2}{3} \times \frac{4}{3} = \frac{8}{9}$$

2. a At the highest point



the composite mass will have both components of its velocity equal.

Hence $\theta = 45^\circ$

3. a $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$... (i)

as $\mu = \frac{3}{2}$ $f = 2R$

$v = 8m$

$u = -24m$

solving equation (i)

$f = 6m \Rightarrow R = 3m$

4. b Lease count = 1 M.S.D. – 1 V.S.D.

$$= (0.05 - 0.049) \text{ cm}$$

$$= 0.001 \text{ cm}$$

$$\text{Diameter} = 5.10 + (0.001) \times 24$$

$$= 5.10 + 0.024$$

$$= 5.124 \text{ cm}$$

5. d The force is radial in nature.

Therefore work done along circular path = 0

6. c $\frac{\Delta l_2}{\Delta l_1} = \frac{l_2}{l_1} \times \frac{A_1}{A_2}$

$$= \frac{1}{2} \times \frac{4}{1} = 2$$

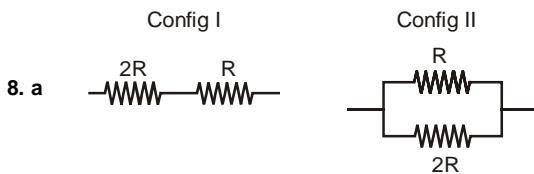
7. a Let θ be the angle between the two vectors

$$\cos \theta = \hat{a} \cdot \hat{b}$$

$$\cos \theta = -\frac{1}{2}$$

$$\theta = 120^\circ$$

$$\text{angle of incidence} = \frac{180 - \theta}{2} = 30^\circ$$



$$\frac{\Delta Q}{\Delta t_1} = \frac{\Delta \theta}{3R}$$

$$\frac{\Delta Q}{\Delta t_2} = \frac{3\Delta \theta}{2R}$$

$$\frac{\Delta t_2}{\Delta t_1} = \frac{2}{9}$$

$$\Delta t_2 = \frac{2}{9} \times 9 = 2 \text{ s}$$

9. b $P = \frac{E}{c} = \frac{30 \times 10^{-3} \times 10 \times 10^{-9}}{3 \times 10^8}$
 $= 1.0 \times 10^{-17} \text{ kg ms}^{-1}$

10. b $I = I_{\max} \cos^2\left(\frac{\phi}{2}\right)$

here $I = \frac{I_{\max}}{2}$

$$\cos\left(\frac{\phi}{2}\right) = \frac{1}{\sqrt{2}}$$

$$\phi = (2n+1)\frac{\pi}{2}$$

or $\Delta x = (2n+1)\frac{\lambda}{4}$

11. a,d Let the mass of lighter sphere be m then the mass of heavier sphere = $3m$

Buoyancy required for equilibrium = $4 mg$

This is only possible when both the spheres are completely submerged.

for the lower sphere

$$kx + 2mg = 3mg$$

$$kx = mg$$

$$x = \frac{4\pi R^3}{3k} \rho g$$



12. b,d In first step,

$$\text{charge on } C_1 = 2CV_0$$

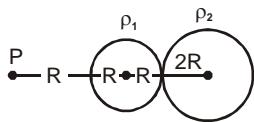
In second step the charge is shared equally between both the capacitors

\therefore charge on C_1 is CV_0 with upper plate positive.

In third step, C_2 gets charged with negative polarity.

\therefore Charge on the upper plate of C_2 is $-CV_0$

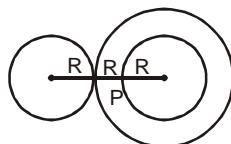
13. b,d Case I



$$\frac{\rho_1 R^3}{3 \epsilon_0 (2R)^2} = \frac{\rho_2 (2R)^3}{3 \epsilon_0 (5R)^2}$$

$$\frac{\rho_1}{\rho_2} = -\frac{32}{25} \quad [\because \text{both must be oppositely charged}]$$

Case II



$$\frac{\rho_1 R^3}{3 \epsilon_0 (2R)^2} = \frac{\rho_2 R}{3 \epsilon_0}$$

$$\frac{\rho_1}{\rho_2} = 4$$

14. b,c No. of nodes = $(m + 1) = 6$

$$\text{Length} = \frac{5\lambda}{2} = \frac{5\pi}{k} = \frac{5 \times 3.14}{62.8} = 0.25 \text{ m}$$

Maximum displacement = 0.01 m

$$\text{Fundamental frequency} = \frac{V}{2L} = \frac{\omega}{2Lk}$$

$$= \frac{628}{2 \times 0.25 \times 62.8} = 20 \text{ Hz}$$

15. a,c The direction of field is $-z$ from right hand rule

$$\text{time } t = \frac{\theta}{\omega}$$

$$\sin \theta = \frac{1}{2}$$

$$\theta = 30^\circ \text{ or } \frac{\pi}{6}$$

$$t = \frac{\pi m}{6 Q B}$$

$$B = \frac{\pi m}{6Q \times 10 \times 10^{-3}} = \frac{50\pi m}{3Q}$$

16. 4

Percentage fraction of sample decayed

$$= (1 - e^{-\lambda t}) \times 100$$

On solving

$$= 4$$

17. 5

At the highest point of mass 1

$$\text{speed} = \sqrt{gl_1}$$

If this speed is sufficient for mass 2 to complete a circle,

$$\sqrt{gl_1} = \sqrt{5gl_2}$$

$$\frac{l_1}{l_2} = 5$$

18. 8

$$\text{Here } l_1\omega_1 = l_2\omega_2 \quad \dots(i)$$

$$l_1 = \frac{1}{2}MR^2 = \frac{1}{2} \times 50 \times 0.4^2 = 4 \text{ units}$$

$$l_2 = \frac{1}{2}MR^2 + 2 \times 2mr^2 = 5 \text{ units}$$

from equation (i)

$$\omega_2 = \frac{4}{5}\omega_1 = 8$$

19. 1

The slope of V vs. v graph is $\frac{h}{e}$ and thus remains same for all
Hence ratio = 1

20. 5

$$W = \frac{1}{2}mv^2$$

$$0.5 \times 5 = \frac{1}{2} \times 0.2 v^2$$

$$v = 5 \text{ m/s}$$

Chemistry

21. a



$$\Delta H = [6 \times (-400) + 6(-300)] - [-1300]$$

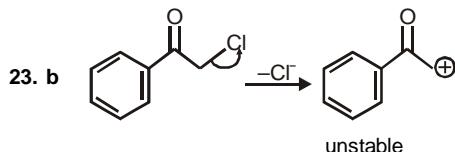
$$= -2400 - 1800 + 1300$$

$$= -4200 + 1300$$

$$\Delta H = -2900 \text{ kJ/mol}$$

$$= \frac{-2900}{180} \text{ kJ/gm}$$

$$= -16.11 \text{ kJ/gm}$$



due to $-I$ effect of $-C=O-$, so the formation of carbocation is not possible. Hence fastest S_N2 is given by S.

24. d Phenol don't liberate CO_2 on reacting with aq. $NaHCO_3$

25. b Spin only magnetic moment $= \sqrt{n(n+2)}$ B.M.

where $n \rightarrow$ no. of unpaired electrons.

$$P = [FeF_6]^{3-} = Fe^{3+} = \sqrt{5(5+2)} = \sqrt{35}$$

$$Q = [V(H_2O)_6]^{2+} = V^{2+} = \sqrt{2(2+2)} = \sqrt{8}$$

$$R = [Fe(H_2O)_6]^{2+} = Fe^{2+} = \sqrt{4(4+2)} = \sqrt{24}$$

26. d w.r.t. 'P' reaction is 1st order
w.r.t. 'Q' reaction is 0 order
overall order = 1 + 0 = 1

27. b HNO_3 on long standing produces NO_2 (Yellow brown)

$$28. a \frac{r}{R} = 0.414; \frac{r}{250} = 0.414$$

$$r = 250 \times 0.414 = 103.5 \text{ pm}$$

$$\square 104 \text{ pm}$$

29. d $Zn + H_2S \rightarrow ZnS + 2H^+$

Curdy white
ppt.

30. b Adsorption (chemical) takes place and releases energy.

31. a,b,c,d

All forms aromatic compounds

32. a $[H^+]$ from weak acid = 10^{-2} M

$$\text{so } C\alpha = 10^{-2}$$

$$K_a = C\alpha^2$$

$$K_a = \frac{(C\alpha)^2}{C} = \frac{(10^{-2})^2}{1} = 10^{-4}$$

33. b,c,d

Being ideal, ΔH and $\Delta S_{\text{surrounding}} = 0$ and entropy of solution on mixing increases.

34. b,d

35. a In tert. carbocation, C has vacant P-orbital and hence $\sigma \rightarrow P$ (empty) overlap takes place.

36. 4

37. 5

$$\frac{\lambda_{He}}{\lambda_{Ne}} = \frac{\sqrt{2mKE_{Ne}}}{\sqrt{2mKE_{He}}} = \frac{\sqrt{2 \times 20} \times \frac{3}{2}KT_{Ne}}{\sqrt{2 \times 4} \times \frac{3}{2}KT_{He}}$$

$$\text{Now } T_{Ne} = 1000 \text{ K}$$

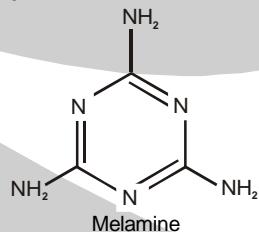
$$T_{He} = 200 \text{ K}$$

$$\text{So } \frac{\lambda_{He}}{\lambda_{Ne}} = \sqrt{25} = 5$$

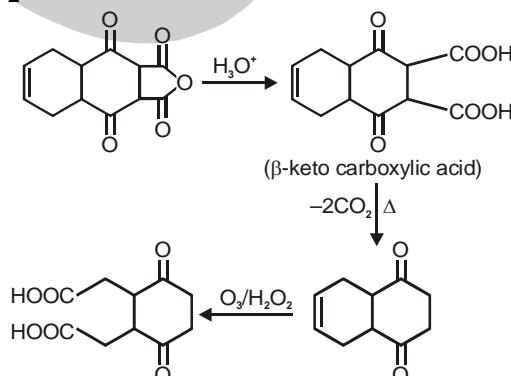
$$\frac{M \times \lambda_{He}}{\lambda_{Ne}} = 5$$

$$M = 5$$

38. 6



39. 2



40. 8

Mathematics

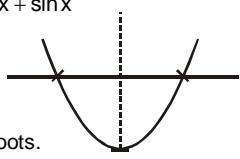
41. c $f(x) = x^2 - x \sin x - \cos x$

$$f'(x) = 2x - \sin x - x \cos x + \sin x$$

$$= x(2 - \cos x)$$

$$f(0) = -2$$

$\therefore f(x) = 0$ have two roots.



42. b $1 + \sum_{k=1}^n 2k = 1 + 2n \frac{(n+1)}{2} = n^2 + n + 1$

$$\sum_{n=1}^{23} \cot^{-1}(n^2 + n + 1)$$

$$\sum \tan^{-1} \frac{n+1-n}{n^2+n+1} = \sum_{n=1}^{23} \tan^{-1}(n+1) - \tan^{-1} n$$

$$= \tan^{-1} 24 - \tan^{-1} 1$$

$$= \tan^{-1} \frac{23}{25}$$

$$\text{Now } \cot \tan^{-1} \left(\frac{23}{25} \right)$$

$$= \cot \cot^{-1} \left(\frac{25}{23} \right) = \frac{25}{23}$$

43. c $\vec{b} + \overline{SQ} = \vec{a}$

$$\vec{b} - \vec{a} = \overline{SQ}$$

$$\vec{b} + \vec{a} = \overline{PR}$$

$$\overline{PS} = \vec{b} = \frac{\overline{SQ} + \overline{PR}}{2} = 2\hat{i} - \hat{j} - 3\hat{k}$$

$$\overline{PQ} = \vec{a} = \frac{\overline{PR} - \overline{SQ}}{2} = \hat{i} + 2\hat{j} + \hat{k}$$

$$\therefore \text{Volume} = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 2 & 1 \\ 2 & -1 & -3 \end{vmatrix} = |1(-6+1) - 2(-3-2) + 3(-1-4)|$$

$$= |-5 + 10 - 15| = 10$$

44. c $|z - z_0| = r$

$$\Rightarrow |z_0|^2 - (\alpha \bar{z}_0 + \bar{\alpha} z_0) + |\alpha|^2 - r^2 = 0 \quad \dots(i)$$

$$\left| \frac{1}{\alpha} - z_0 \right|^2 = 4r^2$$

$$\Rightarrow 1 - (z_0 \bar{\alpha} + \alpha \bar{z}_0) + |\alpha|^2 |z_0|^2 - 4r^2 |\alpha|^2 = 0 \quad \dots(ii)$$

Equation (i) - (ii) is

$$1 - (|z_0|^2 + |\alpha|^2 - r^2) + |\alpha|^2 |z_0|^2 - 4r^2 |\alpha|^2 = 0$$

$$\text{Since } |z_0|^2 = \frac{r^2 + 2}{2}$$

$$\therefore 2 - (r^2 + 2 + 2|\alpha|^2 - 2r^2) + |\alpha|^2 (r^2 + 2) - 8r^2 |\alpha|^2 = 0$$

$$\Rightarrow |\alpha|^2 = \frac{r^2}{7r^2} = \frac{1}{7}$$

$$\text{Hence } |\alpha| = \frac{1}{\sqrt{7}}$$

45. a Point of intersection of $ax + by + c = 0$ and $bx + ay + c = 0$ is

$$\left(-\frac{c}{a+b}, -\frac{c}{a+b} \right)$$

$$\text{Now } \sqrt{\left(\frac{c}{a+b} + 1 \right)^2 + \left(\frac{c}{a+b} + 1 \right)^2} < 2\sqrt{2}$$

$$\sqrt{2} \left| \frac{a+b+c}{a+b} \right| < 2\sqrt{2}$$

$$\Rightarrow \frac{a+b+c}{a+b} < 2$$

$$\Rightarrow 2a + 2b > a + b + c$$

$$\text{i.e. } a + b - c > 0$$

46. d Any point B on the line is $(2\lambda - 2, -\lambda - 1, 3\lambda)$

Point B lies on plane for some λ

$$\Rightarrow (2\lambda - 2) + (-\lambda - 1) + (3\lambda) = 3$$

$$\therefore \lambda = \frac{3}{2} \Rightarrow B = \left(1, -\frac{5}{2}, \frac{9}{2} \right)$$

The foot of perpendicular from point $(-2, -1, 0)$ on the plane is the point A $(0, 1, 2)$

$$\therefore \text{D.R. of AB} \equiv (2, -7, 5)$$

$$\text{Hence equation of line is } \frac{x}{2} = \frac{y-1}{-7} = \frac{z-2}{5}$$

47. a Required prob. $1 - P(\bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D})$

$$= 1 - P(\bar{A})P(\bar{B})P(\bar{C})P(\bar{D})$$

$$= 1 - \frac{1}{2} \times \frac{1}{4} \times \frac{3}{4} \times \frac{7}{8}$$

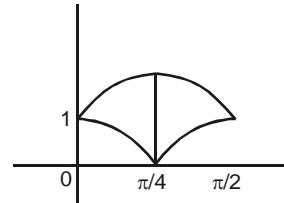
$$= 1 - \frac{21}{256} = \frac{235}{256}$$

48. b $y = \sin x + \cos x$

$$= \sqrt{2} \sin \left(x + \frac{\pi}{4} \right)$$

and $y = |\cos x - \sin x|$

$$= \sqrt{2} \left| \cos \left(x + \frac{\pi}{4} \right) \right|$$



$$\text{Area enclosed} = \int_0^{\pi/4} \{(\sin x + \cos x) - (\cos x - \sin x)\} dx$$

$$+ \int_{\pi/4}^{\pi/2} \sin x + \cos x + (\cos x - \sin x) dx$$

$$= 2(-\cos x) \Big|_0^{\pi/4} + 2(\sin x) \Big|_{\pi/4}^{\pi/2}$$

$$= 2 \left(1 - \frac{1}{\sqrt{2}} \right) + 2 \left(1 - \frac{1}{\sqrt{2}} \right)$$

$$= 4 - 2\sqrt{2} = 2\sqrt{2}(\sqrt{2} - 1)$$

49. a $\frac{dy}{dx} = \frac{y}{x} + \sec\left(\frac{y}{x}\right)$

Let $y = vx$

$$v + x \frac{dv}{dx} = v + \sec v$$

$$\Rightarrow \int \cos v \, dx = \int \frac{dx}{x}$$

$$\sin v = \ln x + \ln c$$

$$\text{or } \sin \frac{y}{x} = \ln x + \ln c$$

$$\sin \frac{\pi}{6} = 0 + \ln c = \frac{1}{2}$$

$$\Rightarrow \sin \frac{y}{x} = \ln x + \frac{1}{2}$$

50. d $f'(x) < 2f(x)$

$$\int_{1/2}^x \frac{f'(x)}{f(x)} dx < \int_{1/2}^x \frac{dx}{2}$$

$$\Rightarrow (\log f(x))_{1/2}^x < 2(x)_{1/2}^x$$

$$\Rightarrow \log f(x) < 2x - 1$$

$$\Rightarrow f(x) < e^{2x-1}$$

$$\Rightarrow 0 < \int_{1/2}^1 f(x) dx < \int_{1/2}^1 e^{2x-1} dx$$

$$0 < \int_{1/2}^1 f(x) dx < \left(\frac{e^{2x-1}}{2} \right)_{1/2}^1$$

$$0 < \int_{1/2}^1 f(x) dx < \frac{e-1}{2}$$

51. c,d (a) If M is symmetric $\Rightarrow M^T = M$

$$\therefore (N^T M N)^T = N^T M^T N = N^T M N = \text{symmetric}$$

If M is skew symmetric $\Rightarrow M^T = -M$

$$(N^T M N)^T = N^T M^T N = N^T (-M) N = -N^T M N$$

\Rightarrow skew symmetric

\therefore option (a) is correct statement.

(b) $(MN - NM)^T = (MN)^T - (NM)^T$

$$= N^T M^T - M^T N^T$$

$$= NM - MN = \text{skew symmetric}$$

\therefore statement (b) is correct

$$(c) (MN)^T = N^T M^T$$

$= NM$, which is not correct

(d) adj(MN) = (adj N)(adj M)

\Rightarrow which is not correct.

52. b,d $L_1 = 3\hat{i} - \hat{j} + 4\hat{k} + t(\hat{i} + 2j66 + 2\hat{k})$

$$L_2 = 3\hat{i} + 3\hat{j} + 2\hat{k} + s(2\hat{i} + 2\hat{j} + \hat{k})$$

Vector \perp to L_1 and L_2 is given by

\hat{i}	\hat{j}	\hat{k}
1	2	2
2	2	1

$$-2\hat{i} + 3\hat{j} - 2\hat{k}$$

Equation of line L is $\frac{x}{-2} = \frac{y}{3} = \frac{z}{-2} = r$

$$x = -2r, y = 3r, z = -2r$$

Putting in L_1 i.e. $\frac{x-3}{1} = \frac{y+1}{2} = \frac{z-4}{2}$

$$-2r - 3 = \frac{3r + 1}{2} = \frac{-2r - 4}{2} \Rightarrow r = -1$$

Intersection of L and L_1 is $x = 2, y = -3, z = 2$

$$\text{point on } L_2 \text{ i.e. } \frac{x-3}{2} = \frac{y-3}{2} = \frac{z-2}{1} = \lambda$$

$$2\lambda + 3, 2\lambda + 3, \lambda + 2$$

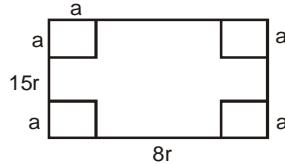
$$\sqrt{(2\lambda + 3 + 2)^2 + (2\lambda + 3 + 3)^2 + (\lambda + 2 - 2)^2} = \sqrt{17}$$

$$\Rightarrow 9\lambda^2 + 28\lambda + 20 = 0$$

$$\Rightarrow \lambda = \frac{-28 \pm 8}{18} = -2, -\frac{10}{9}$$

$$\therefore \text{point on } L_2 \text{ are } (-1, -1, 0) \text{ and } \left(\frac{7}{9}, \frac{7}{9}, \frac{8}{9}\right).$$

53. c,a



$$\Rightarrow 46r = k; k \text{ is constant}$$

$$\text{Also } 4a^2 = 100 \Rightarrow a^2 = 25 \Rightarrow a = 5$$

$$v = (8r - 2a)(15r - 2a) \times a$$

$$v = (120r^2 - 46ar + 4a^2)$$

$$v = 120ar^2 - 46a^2r + 4a^3$$

$$\frac{dv}{da} = 120r^2 - 46r \times 2a + 12a^2 = 0$$

$$\therefore a = 5 \therefore 120r^2 - 460r + 300 = r = 3, \frac{5}{6}$$

$$\therefore \frac{d^2v}{da^2} = -92r + 24a < 0 \text{ for } r = 3$$

\therefore length of sides = $15 \times 3, 8 \times 3$

45, 24.

54. a,d $S_n = \sum_{k=1}^{4n} \frac{k(k+1)}{2}$

$$= 3^2 - 1^2 + 7^2 - 5^2 + 11^2 - 9^2 + 15^2 - 13^2 + \dots (4n-1)^2 - (4n-3)^2$$

$$+ 4^2 - 2^2 + 8^2 - 6^2 + 12^2 - 10^2 + \dots (4n)^2 - (4n-2)^2$$

$$= 8[1 + 3 + 5 + \dots (2n-3) + (2n-1)]$$

$$+ 4[3 + 7 + 11 + \dots (4n-5) + (4n-1)]$$

$$= 8 \times \frac{n}{2} (2 + (n-1)2) + 4 \times \frac{n}{2} [6 + (n-1)4]$$

$$= 16n^2 + 4n$$

$$\text{For } n = 8 \quad S_n = 16 \times 64 + 32 = 1056$$

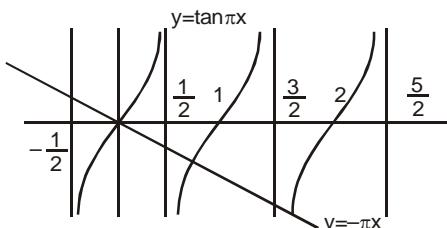
$$n = 9, \quad S_n = 16 \times 81 + 36 = 1332$$

55. b,c $f(x) = x \sin \pi x$

$$f'(x) = \pi x \cos \pi x + \sin \pi x$$

$$f'(x) = 0$$

$$\tan \pi x = -\pi x$$



$$B\left(\frac{3}{2}, 2\right) \text{ unique solution}$$

$$C(1, 2) \text{ unique solution}$$

56.

Let number on remained cards is $k, k+1$

$$\therefore (1+2+\dots+n) - [k+(k+1)] = 1224$$

$$\frac{n(n+1)}{2} - (2k+1) = 1224$$

$$n(n+1) - 4k - 2 = 2448$$

$$n(n+1) - 4k = 2450$$

$$n(n+1) - 2450 = 4k$$

$$n = 50, k = 25$$

$$\therefore k-20 = 5.$$

57.

Eight vectors are

$$\vec{p} = \hat{i} + \hat{j} + \hat{k}$$

$$\vec{p}' = -\hat{i} - \hat{j} - \hat{k}$$

$$\vec{q} = \hat{i} + \hat{j} - \hat{k}$$

$$\vec{q}' = -\hat{i} + \hat{j} + \hat{k}$$

$$\vec{r} = \hat{i} - \hat{j} + \hat{k}$$

$$\vec{r}' = -\hat{i} + \hat{j} - \hat{k}$$

$$\vec{s} = -\hat{i} + \hat{j} + \hat{k}$$

$$\vec{s}' = \hat{i} - \hat{j} - \hat{k}$$

If we take \vec{p} and \vec{p}' and any one of remaining.

six vectors will always be co-planar.

$$\therefore \text{No. of co-planar vectors} = 6$$

Similarly taking q and q' , r and r' and s and s'

No. of co-planar vectors in each case is 6.

$$\therefore \text{Total number of co-planar vectors} = 24$$

\therefore Required number of non-co-planar vector

$$= {}^8C_3 - 24 = 32 - 24 = 8$$

58.

9

Equation of tangent at P

$$\frac{xh}{4} + \frac{yk}{3} = 1$$

$$R\left(\frac{4}{h}, 0\right)$$

60. 6

$$P(E_1 \cap E_2^c \cap E_3^c) = \alpha \Rightarrow P(E_1).P(E_2^c).P(E_3^c) = \alpha \quad \dots(i)$$

$$P(E_1^c \cap E_2 \cap E_3^c) = \beta \Rightarrow P(E_1^c).P(E_2).P(E_3^c) = \beta \quad \dots(ii)$$

$$P(E_1^c \cap E_2^c \cap E_3) = \gamma \Rightarrow P(E_1^c).P(E_2^c).P(E_3) = \gamma \quad \dots(iii)$$

$$P(E_1^c \cap E_2^c \cap E_3^c) = p \Rightarrow P(E_1^c).P(E_2^c).P(E_3^c) = p \quad \dots(iv)$$

from (i) and (iv)

$$\frac{P(E_1)}{P(E_1^c)} = \frac{\alpha}{p} \Rightarrow P(E_1) = \frac{\alpha}{p + \alpha}$$

from (iii) and (iv)

$$\frac{P(E_3)}{P(E_3^c)} = \frac{\gamma}{p} \Rightarrow P(E_3) = \frac{\gamma}{p + \gamma}$$

$$\therefore \frac{P(E_1)}{P(E_3)} = \frac{\alpha(p + \gamma)}{\gamma(p + \alpha)} = \frac{\alpha p + \gamma \alpha}{p \gamma + \alpha \gamma} \quad \dots(v)$$

$$\therefore (\alpha - 2\beta)p = \alpha\beta \Rightarrow \beta = \frac{\alpha p}{\alpha + 2p}$$

$$\text{Also, } (\beta - 3\gamma)p = 2\beta\gamma \Rightarrow \beta = \frac{3\gamma p}{p - 2\gamma}$$

$$\therefore \frac{\alpha p}{\alpha + 2p} = \frac{3\gamma p}{p - 2\gamma}$$

$$\Rightarrow \alpha p = 5\alpha\gamma + 6p\gamma$$

$$\Rightarrow \alpha p + \gamma\alpha = 6(\alpha\gamma + p\gamma)$$

$$\Rightarrow \frac{\alpha p + \gamma\alpha}{\alpha\gamma + p\gamma} = 6$$

∴ from (v)

$$\frac{P(E_1)}{P(E_3)} = 6$$

